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NUMERICAL PREDICTION OF WAVE FORCES ON SUBMERGED HALF-CYLINDERS--ETC(U)
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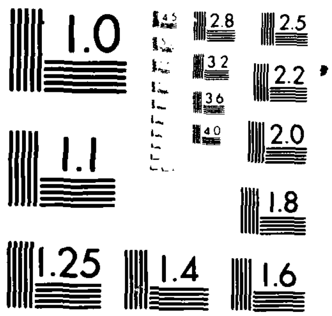

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Numerical Prediction of

Wave Forces

on

Submerged Half-Cylinders

by

C. A. Brebbia

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Introduction

The present report investigates different numerical methods of solution to compute wave forces on submerged shell type structures of arbitrary shape. Although we only dealt here with two dimensional applications, future applications will involve three dimensional analysis. Hence it is of primary importance that the technique under consideration can be extended to three dimensional applications.

The report considers for comparison purpose the application of the finite element method [1] but it is evident that the technique can not be easily extended to general three dimensional problems. Thus we concentrate in the boundary element method [2] [3] **which is newly developed** technique.

The finite element method has attracted the attention of the analysts to largely due to its property of dividing the continuum into a series of elements, which can be associated with physical parts. The existing literature or finite elements is by now very extensive and encompasses structural [4] as well as fluid flow [1] and other types of problems. The method can sometimes be based on variational principles or more generally on weighted residual expressions. Integral equations techniques on the other hand were until recently considered to be a different type of analytical method, somewhat unrelated to other approximate techniques such as finite elements. They become popular in Europe through the work of a series of Russian authors such as Muskhelishvili [5], Mikhlin [6], Kupradze [7]; Smirnov [8] but were not very popular with engineers. A predecessor of some of this work was Kellog [9] who applied integral equations for the solution of Laplace's type problems. Integral equations techniques were mainly used in fluid mechanics and general potential problems and known as the 'source' method, which is now called an 'indirect' method, i.e. the unknowns are not the physical variables of the problem. Work on this method continued throughout the sixties and seventies in the pionerring work of Jaswon [10], and Symm [11], Massonet [12], Hess [13] and many others.

It is difficult to point out precisely who was the first one to propose the 'direct' method of analysis. It is found in a different form, in Kupradze's book [7]. It seems fair however from the engineering point of view to consider that the method originated with the work of Cruse and

Rizzo [14] in elastostatics. The importance of the direct technique is not only that it allows us to solve the problem in terms of physical variables, but also that it was the first step towards a better understanding of the technique and its relationship to other approximate methods, in particular mixed finite element models, such as those proposed by Pain [15].

Since the early 1960's a small research group at Southampton University started working on the application of integral equations to solve engineering problems. Hadid's thesis published in 1964 [16] dealt with the applications of integral equations in shell analysis. Unfortunately the presentation of the problem, the difficulty of defining the appropriate Green's functions and the parallel emergence of the finite element method all contributed to minimize the importance of this work. It was not until 1973 that the group produced another significant thesis, i.e. the one written by Watson [17] and dealing with three dimensional elastostatics problems. By then recent developments in finite elements had started to find their way into the formulation of boundary integral equations. The idea of using general curved elements originated in another Southampton University thesis presented by Lachat in 1975. This thesis whose topic was suggested and supervised by C. Brebbia marked emergence of the boundary element method. Still the question of how to effectively relate the boundary integral equations to other approximate techniques was still unresolved. This was done by Brebbia who published a series of papers on the relationship of different approximate methods, eliminating in 1978 with the first book [2] for which the title Boundary Elements was used. More recently this work has been expanded to encompass time dependent and non-linear problems and this gave origin to a more advanced second boundary elements book [3]. Two important International Conferences were held at Southampton University in 1978 and 1980. The edited proceedings of these conferences - so far the only ones on the topic - are now standard references [18][19].

Although boundary elements is a very powerful technique it is convenient in applications such as fluid-shell interaction problems to combine it with finite elements. This combination should be achieved satisfying fully compatibility and equilibrium at the interfaces between fluid and solid, which requires using the same types of elements for both solutions. The coupling may be achieved in either of two ways; i) by considering

the whole problem using an equivalent BEM approach; ii) by converting the BEM regions into equivalent FEM. The two approaches are described in detail in reference [20]. As the FEM is very well established, the consideration of the BE subregion as an additional equivalent finite element matrix seems most attractive. The formulation of this 'equivalent' matrix used to model the BE presents however, certain problems; at sharp geometric discontinuities, there are also discontinuities of surface tractions which require special attention and the equivalent FE matrix formed is not inherently symmetric, unlike the classical FE approach. A technique which overcomes these problems and provides an acceptable FE type formulation using the BEM method has recently been presented by Georgiou and Brebbia. An early paper involving a crude way of symmetrizing the matrices has been published by Zienkiewicz [22] et al, but this work has been largely superceded by a recent paper by Mustoe et al [23], who presented an interesting way of overcoming some of the difficulties due to the non-symmetric way in which the integrations are carried out. It involves weighting the standard integral equation relationships with the actual shape functions used for the boundary variables. This in effect means that instead of applying a point source to form each equation a distributed source on either side of the node is applied and a numerical integration process which seems the "influence" for a series of sources as opposed to a single one has to be applied. Unfortunately this significantly increases the number of integrations involved. The combination of the solution for the fluid domain, expressed using BEM, the shell discretization using FEM will be the topic of a future progress report.

The present report deals with the accurate computation of the forces on the semimerged shells and concludes that it is necessary to properly define the geometry of the obstruction. Otherwise an uneconomic number of fluid boundary elements is required to obtain accurate solutions. Higher order elements as the one advocated in this report are also necessary to maintain compatibility of shell and fluid movements.

2. Governing Equations

The following equations correspond to two dimensional, incompressible and irrotational flow. The linear wave theory potential for constant depth and without the obstruction can be written as (figure 1)

$$\Phi_o(x,y,t) = -\frac{iga_o}{\omega}\frac{\cosh[\kappa(y+h)]}{\cosh \kappa h} e^{i\kappa x} e^{-i\omega t} \tag{1}$$

where the subscript 'o' refers to the incident field, x is the coordinate in the direction of the wave, y the vertical coordinate measured from the mean water level. g is gravity, a_o is the wave amplitude of the incident wave, ω and κ the wave frequency and number respectively.

This wave potential can be differentiated in the direction of the normal to the boundaries of the domain to give,

$$q_o(x,y) = \frac{ga_o\kappa}{\omega}\left[n_x \frac{\cosh[\kappa(y+h)]}{\cosh \kappa h} - i\, n_y \frac{\sinh[\kappa(y+h)]}{\cosh \kappa h}\right] e^{i\kappa x}$$

$$Q_o(z,y,t) = q_o(x,y)e^{-i\omega t} \tag{2}$$

where n_x and n_y are the direction cosines of the normal to the boundary with respect to x and y.

The potential of the scattered wave can now be written as,

$$\Phi_s(x,y,t) = \phi_s(x,y)e^{-i\omega t} \tag{3}$$

The total potential is

$$\Phi = \Phi_o + \Phi_s \tag{4}$$

and have to satisfy the Laplace equation in the domain, i.e. (figure 2)

$$\nabla^2\Phi(x,y,t) = 0 \qquad \text{in } \Omega \tag{5}$$

with the following b-c

i) Bottom condition,

$$\frac{\partial \Phi}{\partial n} = 0 \qquad \text{on } \Gamma_2 \tag{6}$$

ii) Free surface condition

$$\frac{\partial^2 \Phi}{\partial t^2} + g \frac{\partial \Phi}{\partial y} = 0 \qquad \text{on } \Gamma_4 \tag{7}$$

iii) Obstruction condition

$$\frac{\partial \Phi}{\partial n} = 0 \qquad \text{on } \Gamma_1 \tag{8}$$

iv) Radiation - Summerfeld type - condition

$$\lim_{|x| \to \infty} \left| \frac{\partial \Phi_s}{\partial |x|} - \frac{\partial \Phi_s}{\partial t} \right| = 0 \qquad \text{on } \Gamma_3 \tag{9}$$

The Φ_o solution for the incident wave will satisfy these conditions, hence the problem can be expressed in terms of the scattered field carrying out the time derivatives. This gives the following system

$$\nabla^2 \phi_s(x,y) = 0 \quad \text{in } \Omega \tag{10}$$

with the following boundary conditions

i) Bottom condition,

$$\frac{\partial \phi_s}{\partial n} = 0 \qquad \text{on } \Gamma_2 \tag{11}$$

ii) Free surface condition

$$\frac{\partial \phi_s}{\partial n} - \frac{\omega^2}{g} \phi_s = 0 \qquad \text{on } \Gamma_4 \tag{12}$$

iii) Obstruction condition

$$\frac{\partial \phi_s}{\partial n} = - \frac{\partial \phi_o}{\partial n} = - q_o \qquad \text{on } \Gamma_1 \tag{13}$$

iv) Radiation condition (approximate)

$$\frac{\partial \phi_s}{\partial n} - i\kappa\phi_s = 0 \qquad \text{on } \Gamma_3 \tag{14}$$

3. Weighted Residual Statement

The above system of equations can now be rewritten in a weighted residual form, to minimize errors when using an approximate method of solution such as finite or boundary elements. Let us consider an arbitrary function ϕ^* and its derivative $q^* = \frac{\partial \phi^*}{\partial n}$. Later on we will associate these functions with the virtual increment type of functions used in finite elements or with the fundamental solutions of boundary elements. The resulting weighted residual statement can be written

$$\int (\nabla^2\phi)\phi^* d\Omega = \int_{\Gamma_1} \left(\frac{\partial \phi}{\partial n} + q_o\right)\phi^* d\Gamma + \int_{\Gamma_2} \frac{\partial \phi}{\partial n}\phi^* d\Gamma$$

$$+ \int_{\Gamma_3} \left(\frac{\partial \phi}{\partial n} - i\kappa\phi\right)\phi^* d\Gamma + \int_{\Gamma_4} \left(\frac{\partial \phi}{\partial n} - \frac{\omega^2}{g}\phi\right)\phi^* d\Gamma \tag{15}$$

Notice that the scattered potential field function ϕ_s is now written without the subscript 's' for simplicity.

Integrating by parts once the terms on the left hand side of (15) results in,

$$\int_{\Omega} \frac{\partial\phi}{\partial x_k}\frac{\partial\phi^*}{\partial x_k}\, d\Omega = -\int_{\Gamma_1} q_o\ \phi^* d\Gamma + i\int_{\Gamma_3} \kappa\phi\phi^* d\Gamma + \int_{\Gamma_4} \frac{\omega^2}{g}\ \phi\phi^* d\Gamma \qquad (16)$$

This expression is the starting point for finite element solutions of the wave diffraction problem.

If we integrate (16) once more one obtains,

$$-\int_{\Omega} \phi\ \nabla^2\phi^* d\Omega = -\int_{\Gamma_1} q_o\ \phi^* d\Gamma + i\int_{\Gamma_3} \kappa\phi\ \phi^* d\Gamma + \int_{\Gamma_4} \frac{\omega^2}{g}\ \phi\phi^* d\Gamma$$

$$- \int_{\Gamma} \phi\ \frac{\partial\phi^*}{\partial n}\, d\Gamma \qquad (17)$$

Notice that Γ is the total boundary i.e. $\Gamma = \Gamma_1 + \Gamma_2 + \Gamma_3 + \Gamma_4$.

Expression (17) is the starting point for boundary element solutions. We need to define ϕ^* as a fundamental solution such that the integral in Ω disappears and the problem becomes a boundary problem only.

4. Finite Element Formulation

Let us first consider the finite element discretization of equation (16). This formulation will imply to define some type of interpolation functions over an element (figure 3). The ϕ function can now be written as,

$$\phi = \underset{\sim}{N}\ \underset{\sim}{\phi}^n \qquad (18)$$

where $\underset{\sim}{N}$ are the interpolation functions chosen and ϕ^n the corresponding nodal unknowns. The inverted form of (18) is

$$\phi^* = \delta\phi = \underset{\sim}{N}\ \delta\underset{\sim}{\phi}^n \qquad (19)$$

Notice that ϕ is a complex function, i.e.

$$\phi = \varphi + i\psi \tag{20}$$

The resulting finite element matrices can now be deduced as indicated in the literature [1] and the final system of equations for the continuum as shown in figure 4 have the following form,

$$\left\{ \underset{\sim}{\kappa} - (i\, \underset{\sim}{\kappa} + \frac{\omega^2}{g})\, \underset{\sim}{M} \right\} \underset{\sim}{\phi}^n = \underset{\sim}{Q} \tag{21}$$

which can now be solved.

5. Boundary Element Formulation

In the boundary element case the ϕ^* functions can be associated with a full space Green's function such that,

$$\nabla^2\phi^* + \Delta_i = 0 \tag{22}$$

where Δ_i is a Dirac delta function, whose integral is equal to one at the point 'i' and zero everywhere else.

The solution to (22) is

$$\phi^* = \frac{1}{2\pi} \ln \left(\frac{1}{|r|}\right) \tag{23}$$

Substituting (23) into equation (17) gives,

$$\phi_i = -\int_{\Gamma_1} q_o\, \phi^* d\Gamma + \int_{\Gamma_3} i\kappa\phi\phi^* d\Gamma + \int_{\Gamma_4} \frac{\omega^2}{g} \phi\phi^* d\Gamma - \int_{\Gamma} \phi \frac{\partial\phi^*}{\partial n} d\Gamma \tag{24}$$

Formula (24) implies an integral relationship between a point 'i' inside the Ω domain and the values on the Γ boundary of the domain. If the point i is taken to be on the domain we have [2],

$$c_i \phi_i = - \int_{\Gamma_1} q_o \phi\phi^* d\Gamma + \int_{\Gamma_3} i\kappa\phi\phi^* d\Gamma + \int_{\Gamma_4} \frac{\omega^2}{g} \phi\phi^* d\Gamma - \int_{\Gamma} \phi \frac{\partial\phi^*}{\partial n} d\Gamma \quad (25)$$

where $c_i = \frac{1}{2}$ for smooth boundaries and for a sharp corner its value is proportional to the interior angle - c_i can also be deduced from consideration of constant field states, see reference [2] [3].

Equation (25) can be rearranged as follows,

$$c_i \phi_i + \int_{\Gamma_1+\Gamma_2} \frac{\partial\phi^*}{\partial n} \phi \, d\Gamma + \int_{\Gamma_3} \left(\frac{\partial\phi^*}{\partial n} - i\kappa\phi^*\right) \phi \, d\Gamma$$

$$+ \int_{\Gamma_3} \left(\frac{\partial\phi^*}{\partial n} - \frac{\omega^2}{g} \phi^*\right) \phi \, d\Gamma + \int_{\Gamma_1} \phi^* q_o \, d\Gamma = 0 \quad (26)$$

We can now propose a type of element to discretize the Γ surface of the domain (see figure 5). The type of elements is of the utmost importance in the analysis as we will see but for simplicity let us consider that the elements are constant. The surface Γ can now be discretized into N elements each with a Γ_j surface and such that (26) becomes,

$$c_i \phi_i + \sum_{N_1+N_2} \phi_j \left(\int_{\Gamma_j} \frac{\partial\phi^*}{\partial n} d\Gamma\right) + \sum_{N_3} \phi_j \left(\int_{\Gamma_j} \frac{\partial\phi^*}{\partial n} - i\kappa\phi^* \, d\Gamma\right)$$

$$+ \sum_{N_4} \phi_j \left(\int_{\Gamma_j} \left(\frac{\partial\phi^*}{\partial n} - \frac{\omega^2}{g} \phi^*\right) d\Gamma\right) + \sum_{N_1} q_{o_j} \left(\int_{\Gamma_j} \phi^* d\Gamma\right) = 0 \quad (27)$$

Note that the total number of elements is $N_1 + N_2 + N_3 + N_4 = N$ and that the 'j' subscript refers to the element number.

We can distinguish two types of integrals.

$$\hat{h}_{ij} = \int_{\Gamma_j} \left(\frac{\partial \phi^*}{\partial n}\right)_{ij} d\Gamma \tag{28}$$

and

$$g_{ij} = \int_{\Gamma_j} (\phi^*)_{ij} \, d\Gamma$$

These integrals represent influence functions between element i at which the fundamental solution is applied and any other element 'j' under consideration. Note that for the case of linear elements $g_{ii} \equiv 0$, as n and Γ are orthogonal. The value of the h_{ii} coefficient is $h_{ii} = \hat{h}_{ii} + c_i$. These coefficients can now be arranged in matrix form, each row of the matrix corresponding to a new 'i' point. Equation (27) now becomes,

$$\left[\, \underset{\sim}{H}_1 \quad \underset{\sim}{H}_2 \quad \underset{\sim}{H}_3 - i\kappa \underset{\sim}{G}_3 \quad \underset{\sim}{H}_4 - \frac{\omega^2}{g} \underset{\sim}{G}_4 \,\right] \begin{Bmatrix} \underset{\sim}{\Phi}_1 \\ \underset{\sim}{\Phi}_2 \\ \underset{\sim}{\Phi}_3 \\ \underset{\sim}{\Phi}_4 \end{Bmatrix} = - \left[\, \underset{\sim}{G}_1 \,\right] \{ \underset{\sim}{Q}_{o1} \} \tag{29}$$

Notice that H and G matrices are real but $\underset{\sim}{\Phi}$ and $\underset{\sim}{Q}$ are complex.

$$\begin{aligned} \underset{\sim}{\Phi}_n &= \underset{\sim}{\varphi}_n + i\, \underset{\sim}{\psi}_n \\ \underset{\sim}{Q}_{o1} &= \underset{\sim}{R}_1 + i\, \underset{\sim}{S}_1 \end{aligned} \tag{30}$$

Here we can write,

$$\begin{bmatrix} \underset{\sim}{H}_1 & \underset{\sim}{0} & \underset{\sim}{H}_2 & \underset{\sim}{0} & \underset{\sim}{H}_3 & \kappa \underset{\sim}{G}_3 & \underset{\sim}{H}_4 - \frac{\omega^2}{g} \underset{\sim}{G}_4 & 0 \\ \underset{\sim}{0} & \underset{\sim}{H}_1 & \underset{\sim}{0} & \underset{\sim}{H}_2 & -\kappa \underset{\sim}{G}_3 & \underset{\sim}{H}_3 & \underset{\sim}{0} & \underset{\sim}{H}_4 - \frac{\omega^2}{g} \underset{\sim}{G}_4 \end{bmatrix} \begin{Bmatrix} \varphi_1 \\ \psi_1 \\ \varphi_2 \\ \psi_2 \\ \varphi_3 \\ \psi_3 \\ \varphi_4 \\ \psi_4 \end{Bmatrix} = - \begin{Bmatrix} \underset{\sim}{G}_1 \underset{\sim}{R}_1 \\ \underset{\sim}{G}_1 \underset{\sim}{S}_1 \end{Bmatrix} \tag{31}$$

Solution of equation (31) will give the values of scattered potential required to solve the problem.

6. Wave Forces

One can now compute the forces on the obstruction from Bernoulli's equation. The dynamic pressure is

$$p(x,y,t) = -\rho \; \mathrm{Re} \left\{ \frac{\partial \Phi(x,y,t)}{\partial t} \right\} \tag{32}$$

where $\Phi = (\varphi + i\psi)e^{-i\omega t}$. Hence

$$p = -\rho \; \mathrm{Re} \; \{-i\omega(\varphi + i\psi)e^{-i\omega t}\} \tag{33}$$

$$= \rho \; \omega(\varphi \sin\omega t - \psi \cos\omega t)$$

Hence,

$$p = \rho\omega R \sin(\omega t - \alpha)$$

with $R = \sqrt{\varphi^2 + \psi^2}$, $\tan\alpha = \frac{\psi}{\varphi}$.

The forces in the horizontal and vertical directions can now be obtained integrating the pressures, i.e.

$$\vec{F} = -\int_{\Gamma_1} p(-\vec{n})\,d\Gamma \tag{35}$$

$\vec{n}$ is the vector normal to the obstruction; $\vec{n} = (\sin\theta, -\cos\theta)$ with θ defined in figure 6.

$$\vec{F} = \int_{\Gamma_1} p(\sin\theta, -\cos\theta)\,d\Gamma = \left\{ \int_{\Gamma_1} p\,dy, \; - \int_{\Gamma_1} p\,dx \right\} \tag{36}$$

Hence in discretized form.

$$F_x = \int_{\Gamma_1} p\, dy \qquad ; \qquad F_y = - \int_{\Gamma_1} p\, dx \tag{37}$$

$$= \sum_{j=1}^{N1} p_j\, \Delta y_u \qquad\qquad = - \sum_{j=1}^{N1} p_j\, \Delta x_j$$

We could also find moments about a given part $x_o y_o$. This gives,

$$M_o = \int_{\Gamma_1} \{ -p \left[(x-x_o)n_y - (y-y_o)n_x \right] \} d\Gamma \tag{38}$$

7. Numerical Results

The forces on cylindrical obstructions of the type shown in figure 1 were studied for two different depth to radius ratio, i.e. $h/a = 3.0$ and $h/a = 5.0$ and for a range of wave numbers. Results obtained using constant boundary elements were compared against linear finite elements and the results published by Chakrabarti [24].

At first the boundary element results were obtained by fitting the elements trying to follow the curved surface of the cylinder. The results obtained in this way for a mesh of 16 elements and the cylinder were very poor and are not shown in the figures. Further investigation pointed out that the reason for this disagreement was the discontinuity of the incident q function between elements, specially for the elements at the bottom. A fine mesh - of 200 elements - would be required to obtain accurate results using constant elements. This mesh density is unsuitable for three dimensional applications and points out the need of using higher order elements which avoid discontinuities at element corners. The importance of these discontinuities is demonstrated by the reasonable accuracy of the BE solutions shown in figures 7 to 10. Thee solutions were obtained by using constant elements in a step fashion, i.e. the elements joining at 90^o (or multiple of 90^o) angles. This grid, similar to the one on the boundary of a finite difference grid gave accurate results with a very coarse grid.

The finite element results shown in figures 7 to 10 were obtained to compare against boundary elements. It is easy to see that a much larger number of degrees of freedom would be necessary when using finite elements, specially for three dimensional problems.

Figures 11 to 14 compare linear boundary elements with Chakrabarti's results. These elements follow the geometry of the half-cylinder and reasonable agreement is obtained with 46 elements. It was then decided to study the convergence of the results for vertical and horizontal forces Figures 15 and 16 show that the results converge when the number of elements is increased. To improve the rate of convergence one needs to employ quadratic or cubic elements.

Conclusions

Three important conclusions were obtained from this work, namely

i) The advantage of using boundary elements by comparison with finite elements in order to reduce the number of unknowns. For three dimensional problems this reduction is in the order of 10 allowing for the solution of problems which otherwise would be impossible to obtain.

ii) The need of using boundary elements which can follow the geometry of the structure under consideration. In particular it is advisable to use at least quadratic elements for curved surfaces or even cubic elements which insure continuity of slopes. It is important to remember that as in finite elements, the functions for the potential should be at least of the same degree as those used for the geometry. Otherwise appreciable errors can be introduced in the formulation [3].

iii) Future work will involve the development of a cubic boundary element to make the potential fully compatible with the type of cubic functions used for shells.

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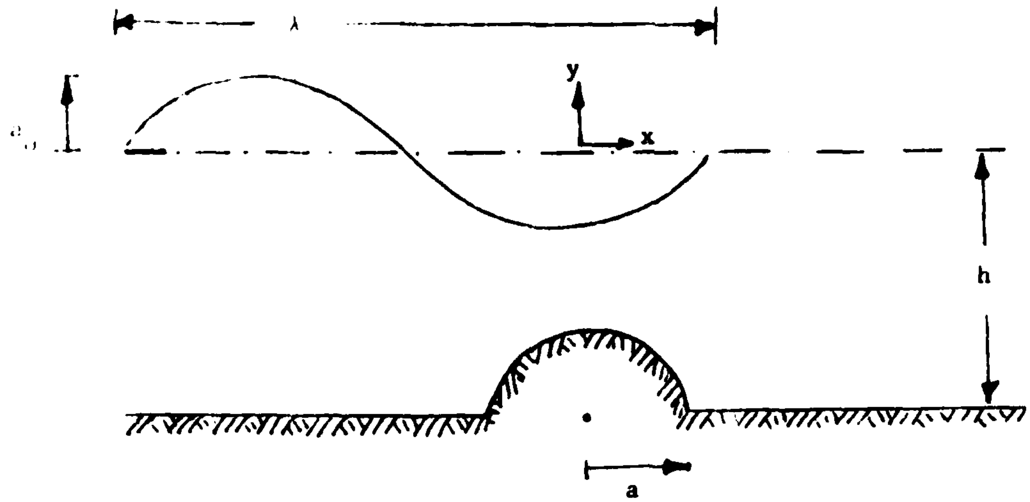


$$\kappa = \frac{2\pi}{\lambda} \qquad \omega = \frac{2\pi}{T}$$

Figure 1 Problem definition

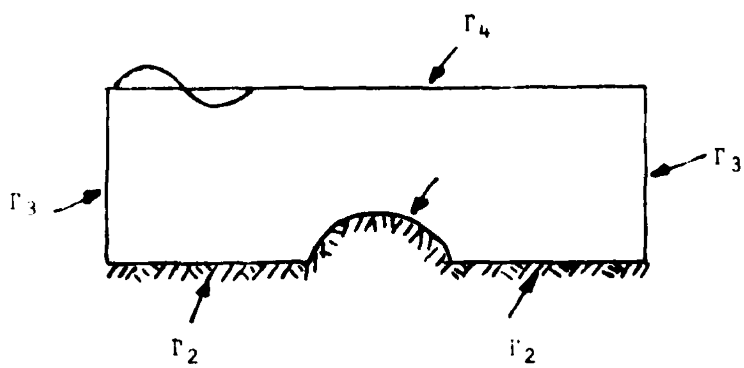


Figure 2 Types of Boundaries

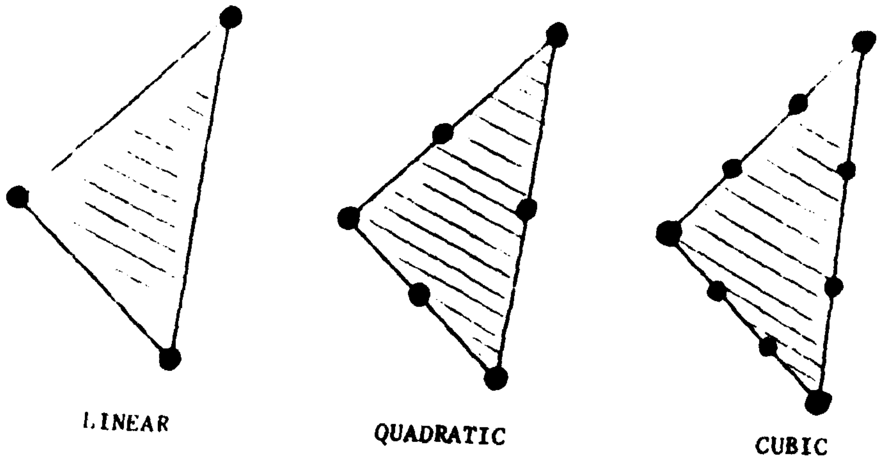


Figure 3 Finite Elements

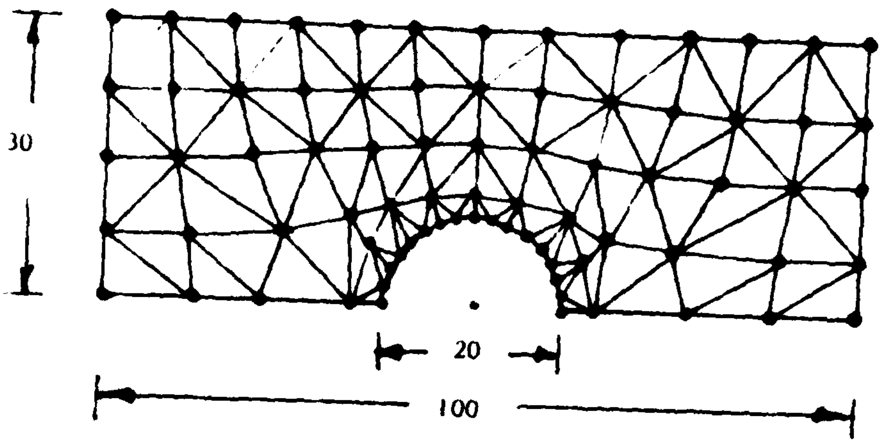


Figure 4 Finite Element Mesh

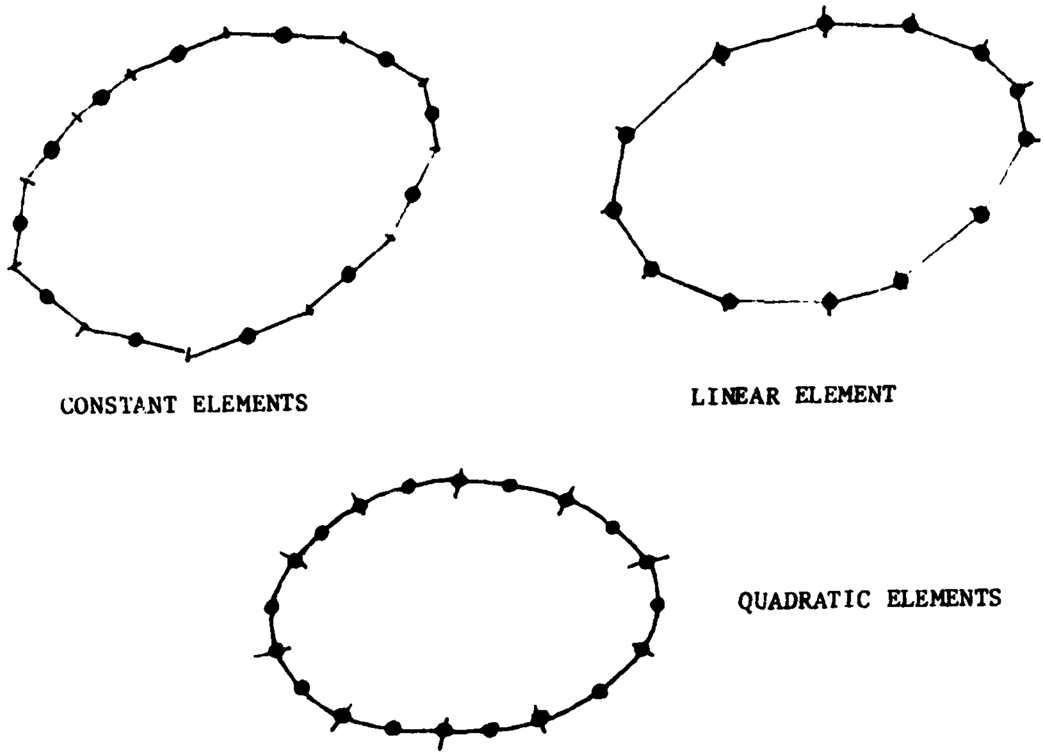


Figure 5 Element Types

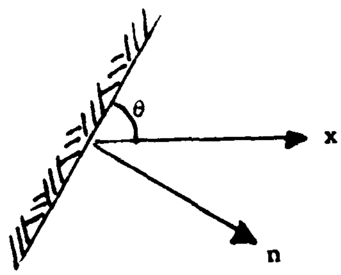


Figure 6 Definition of the Normal

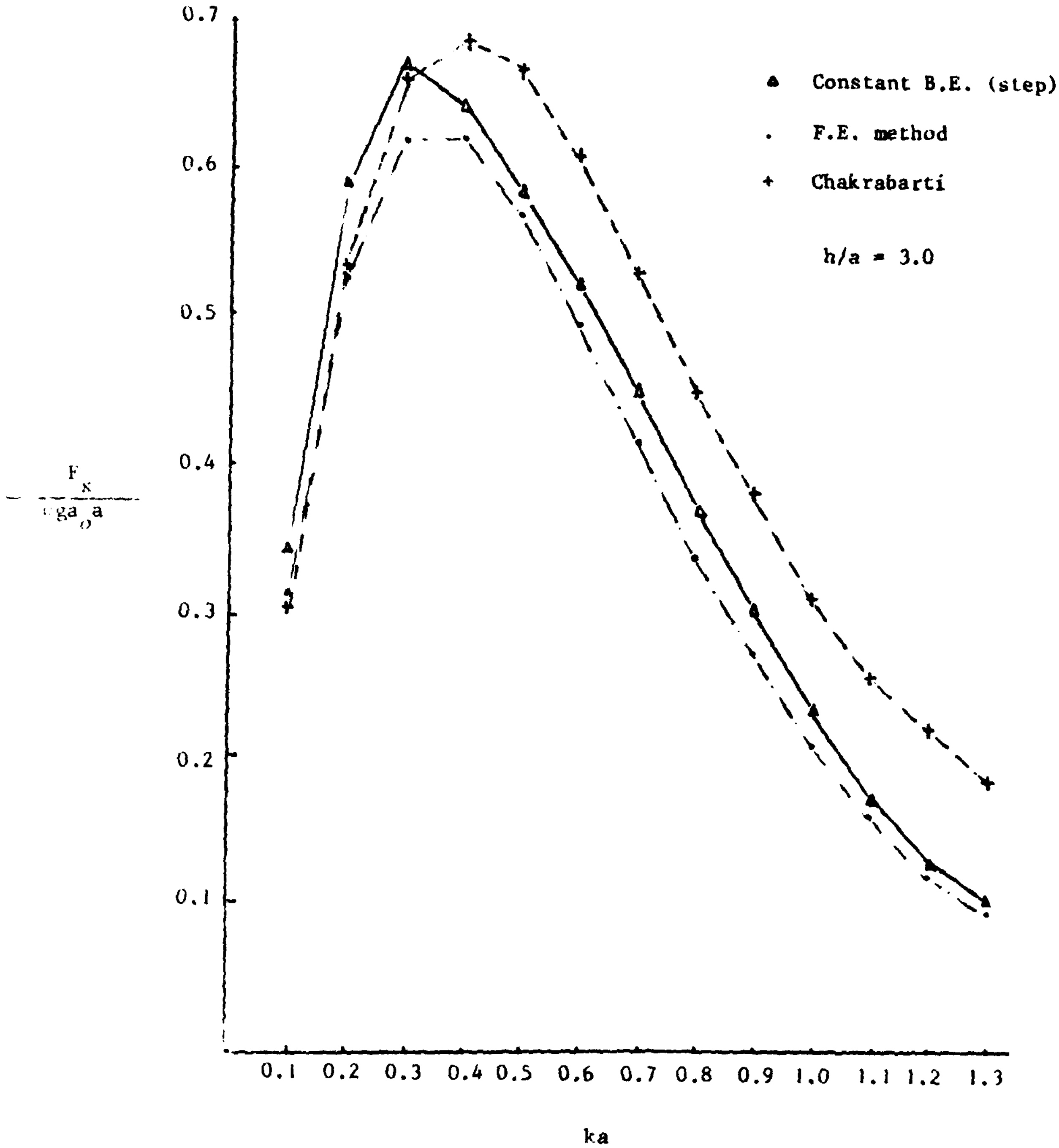


Figure 7 Horizontal Forces on Half-Cylinder
h/a = 3.0

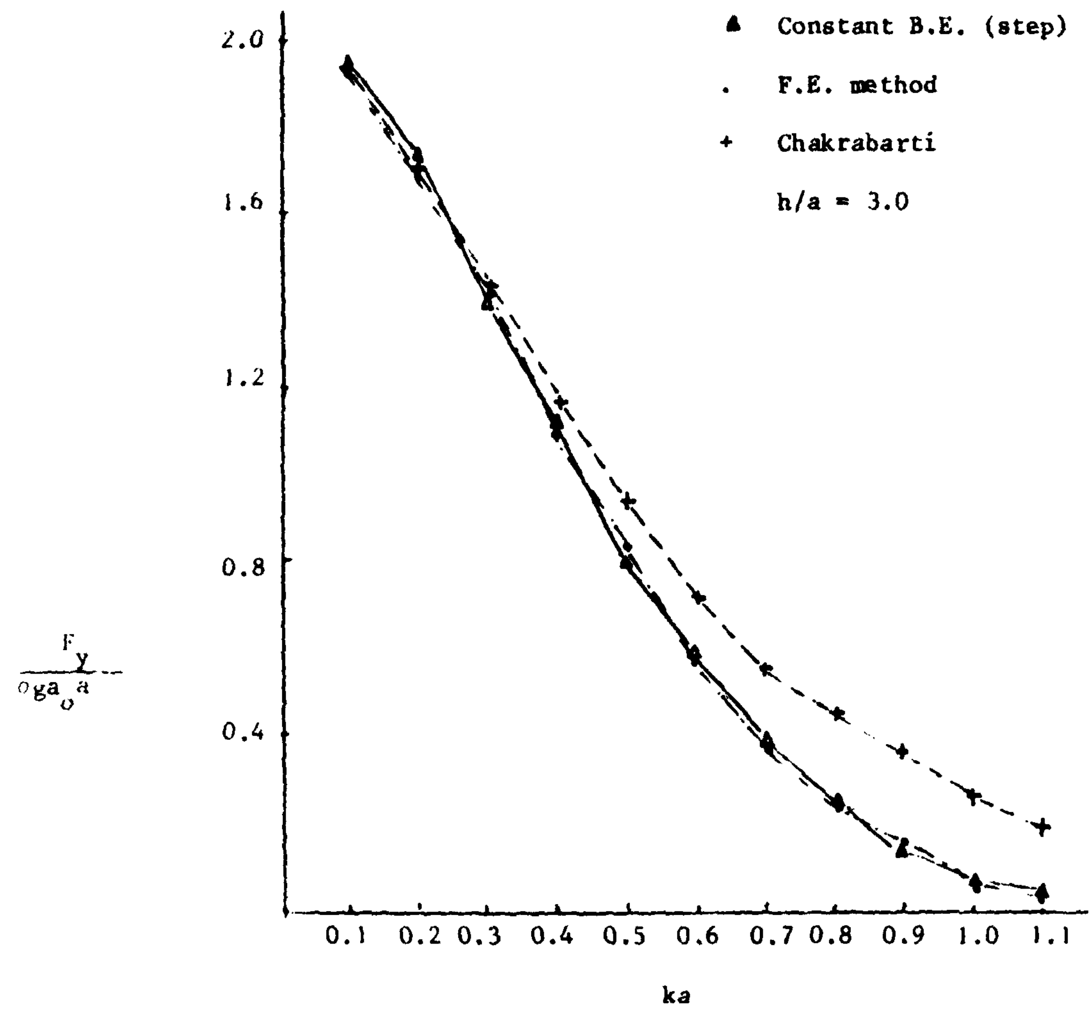


Figure 8 Vertical Forces on Half Cylinder
h/a = 3.0

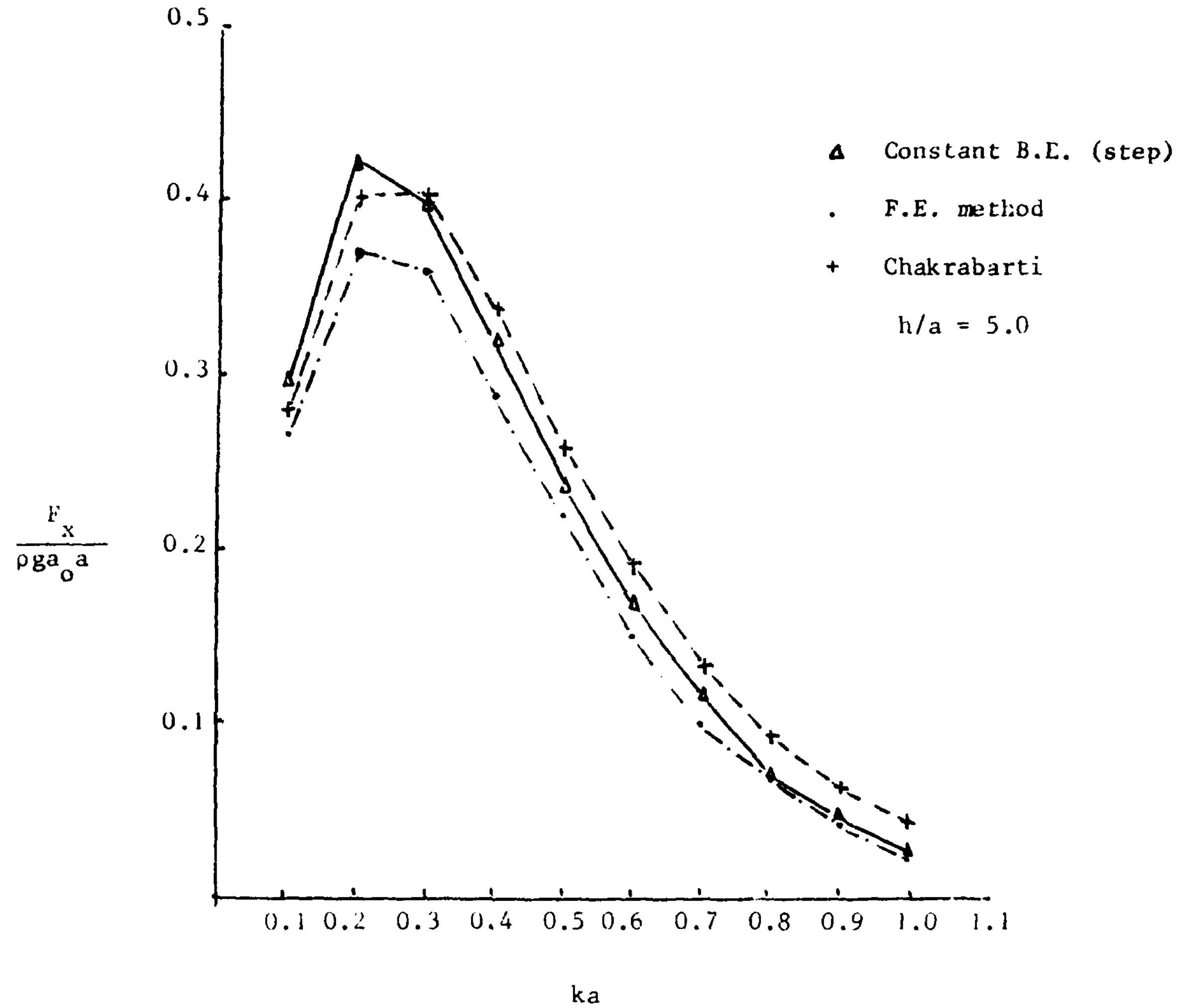


Figure 9 Horizontal Forces on Half-Cylinder
h/a = 5.0

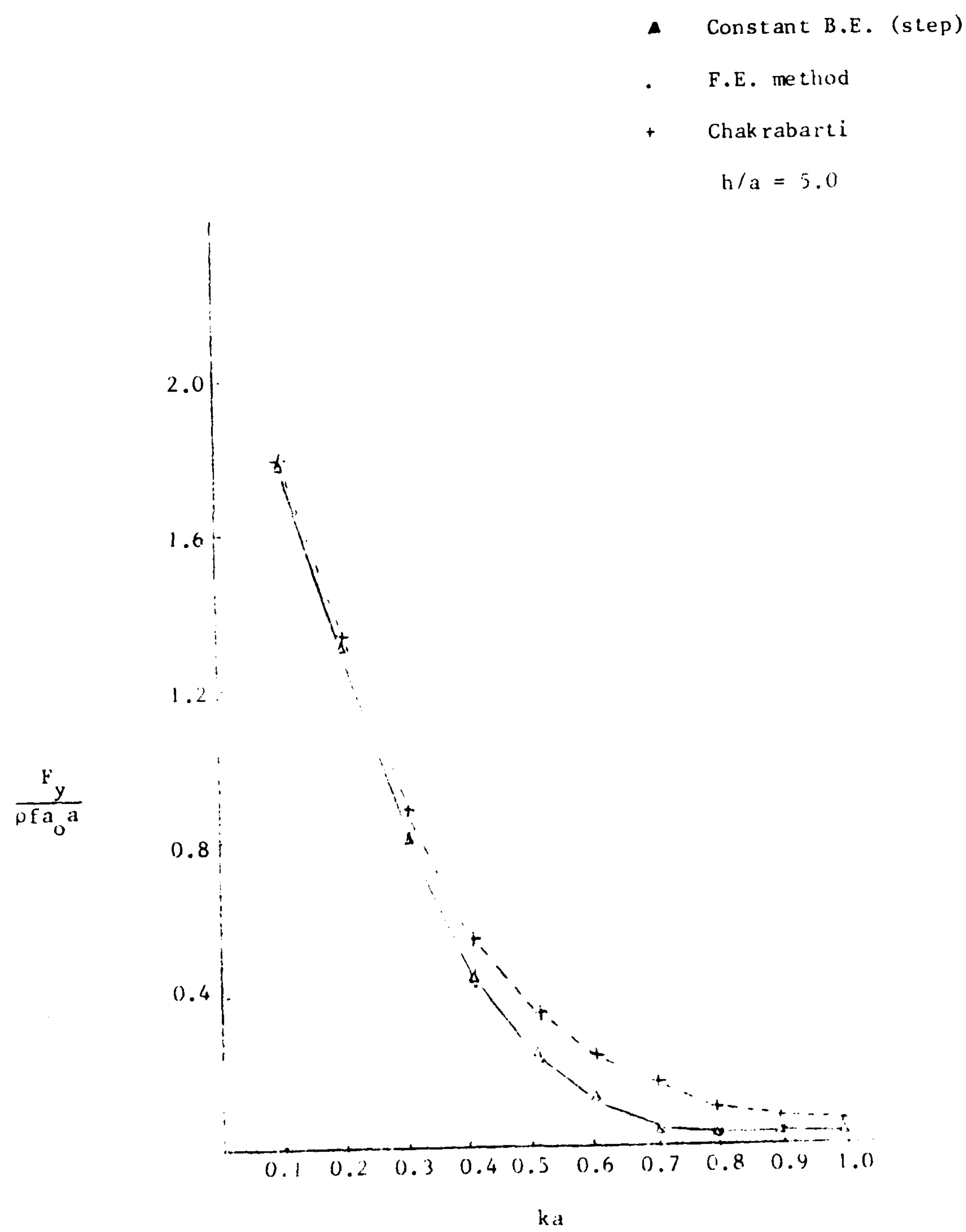


Figure 10 Vertical Forces on Half-Cylinder

h/a = 5.0

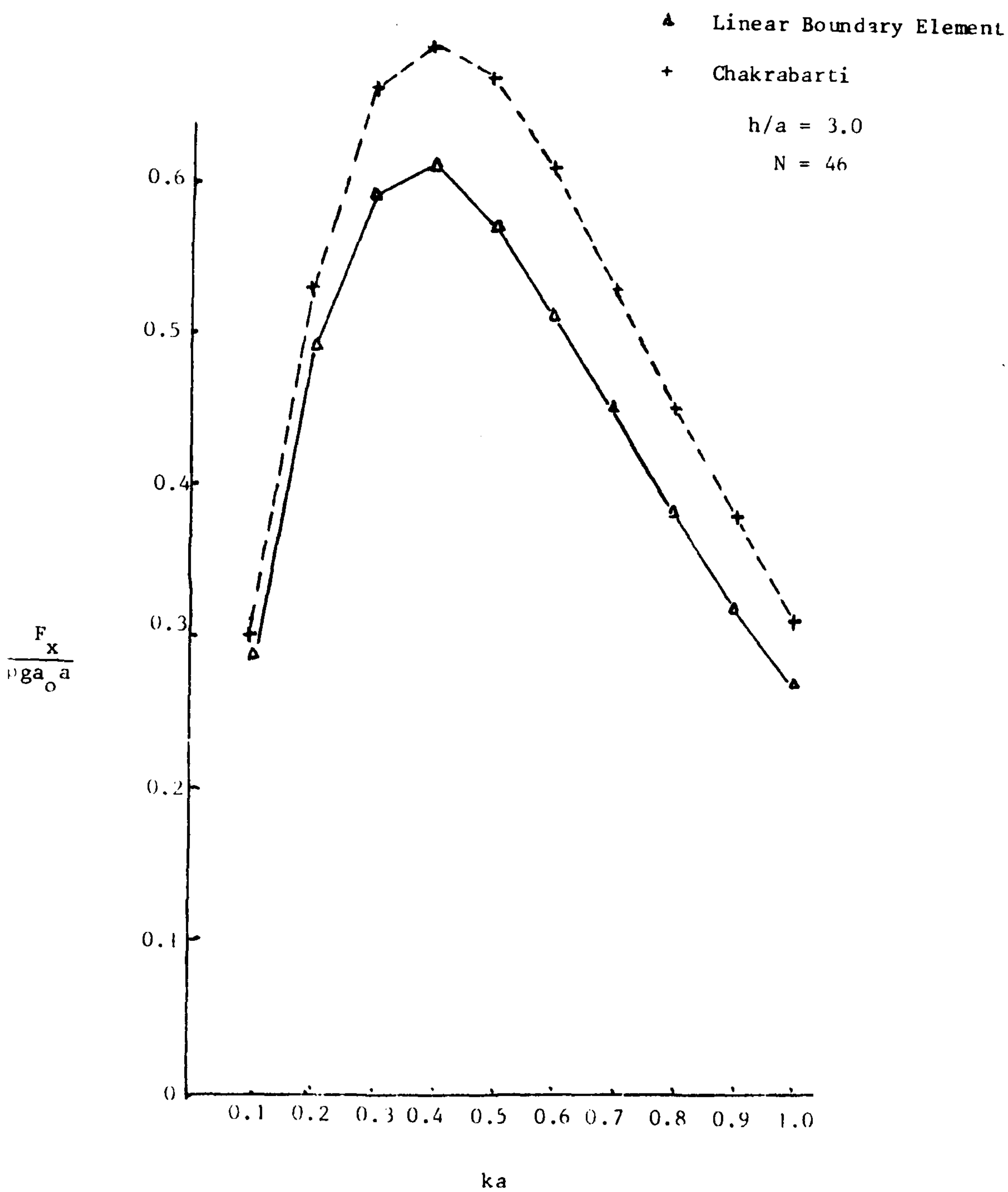


Figure 11 Horizontal Forces on Half-Cylinder

h/a = 3.0

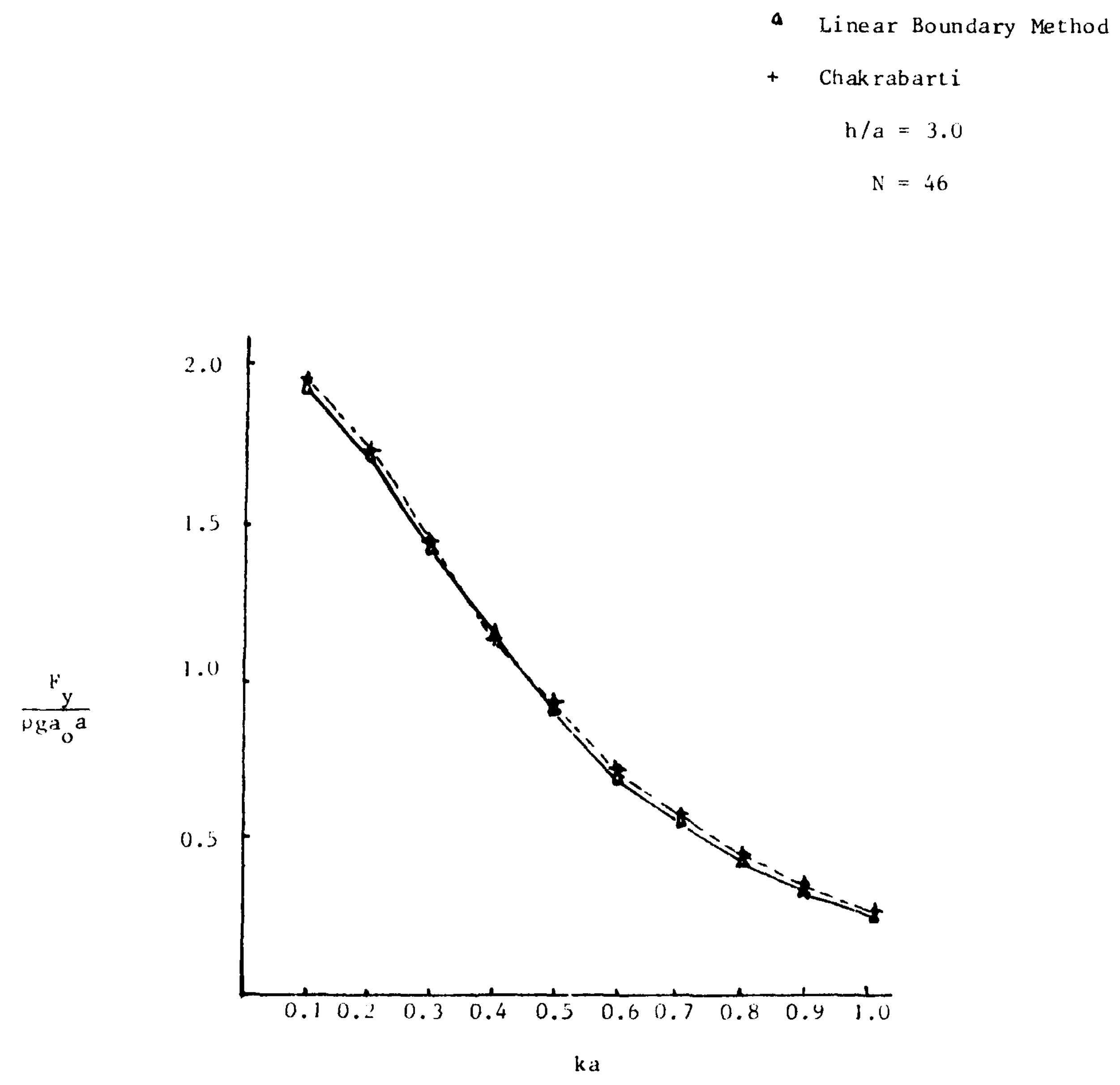


Figure 12 Vertical Forces on Half-Cylinder
h/a = 3.0

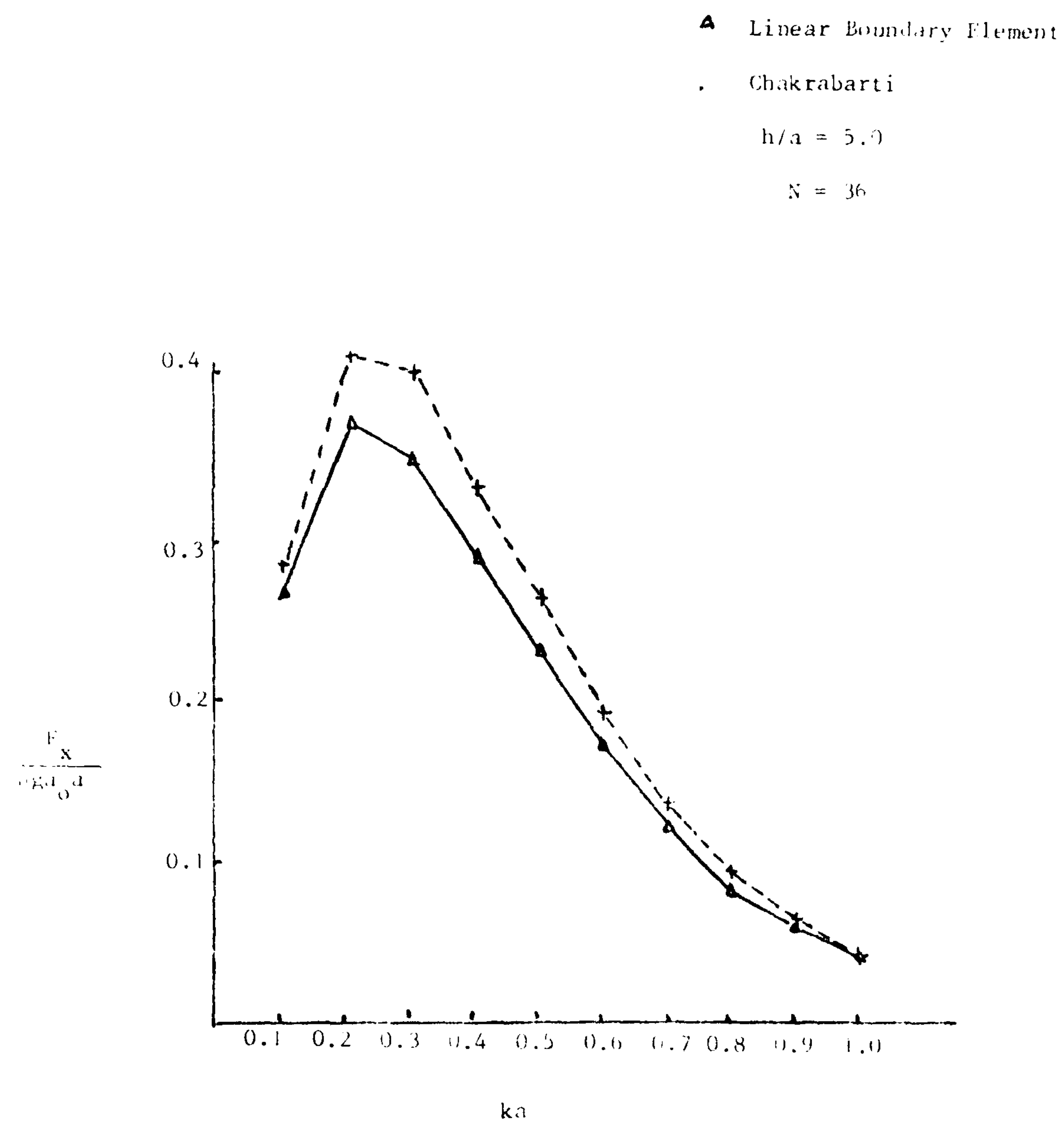


Figure 13 Horizontal Forces on Half-Cylinder

h/a = 5.0

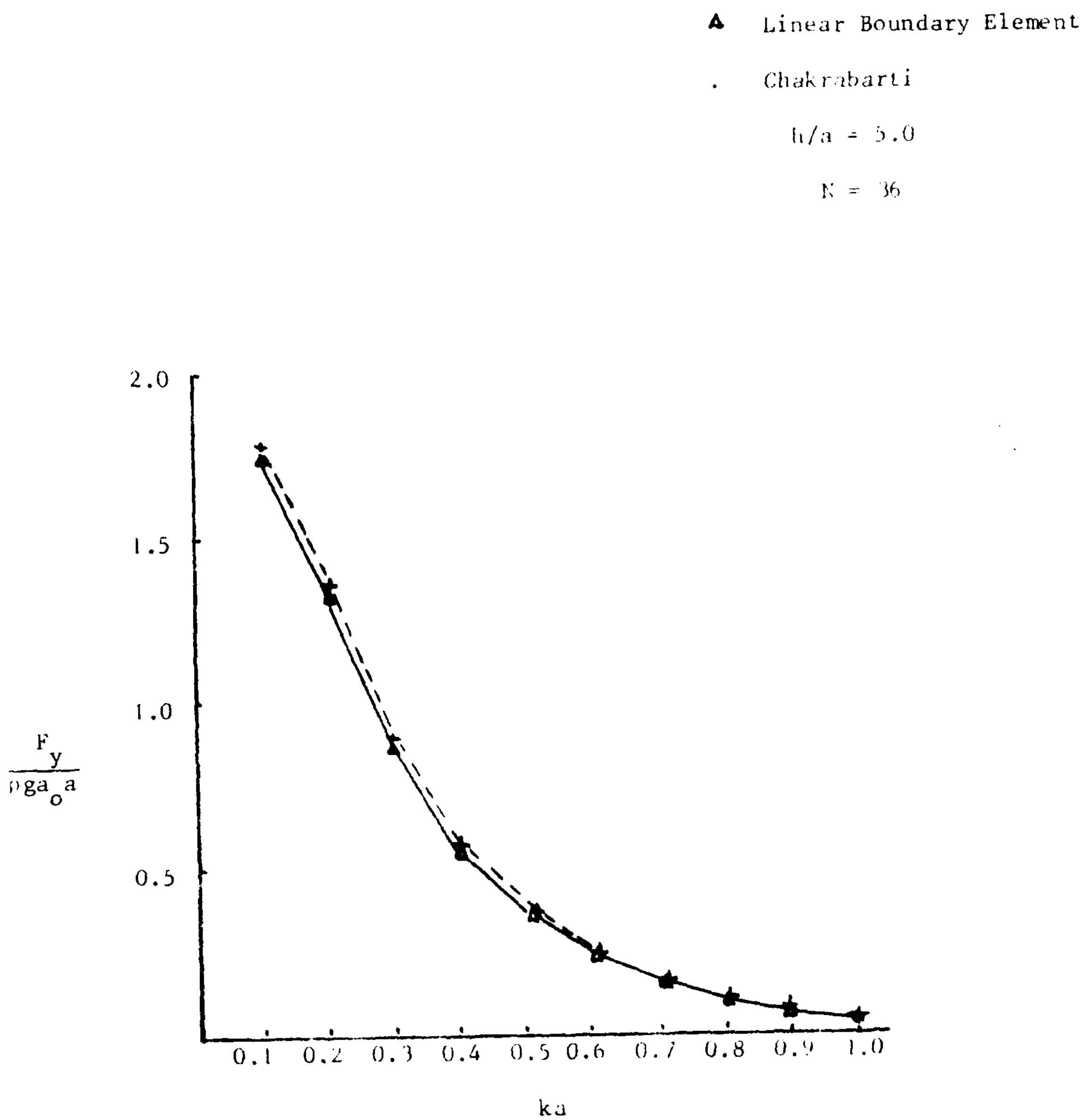


Figure 14 Vertical Forces on Half-Cylinders

h/a = 5.0

Figure 15 Convergence of the horizontal force
(Linear Boundary Elements)

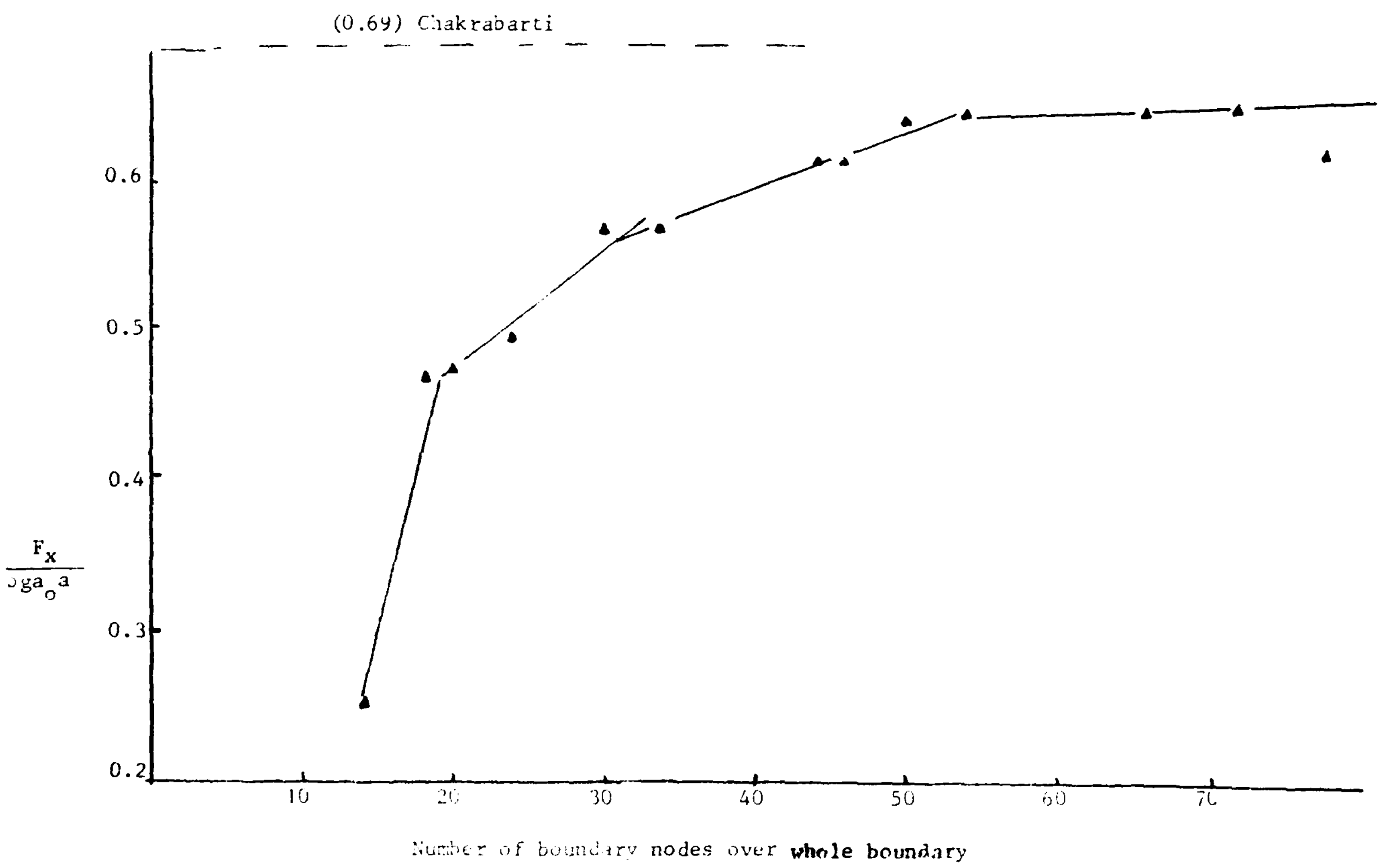
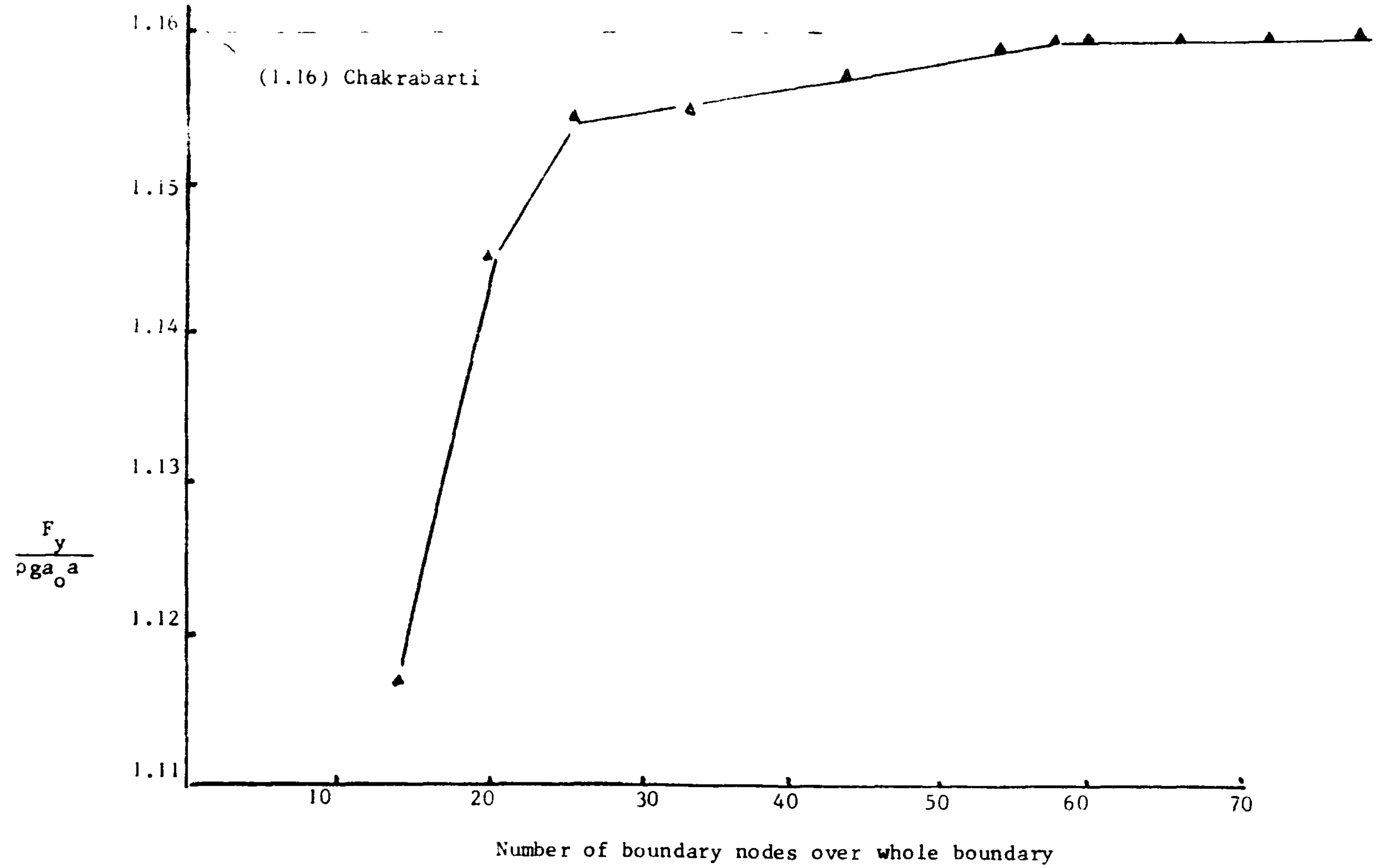


Figure 16 Convergence of the vertical force (Linear Boundary Elements)



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